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BUBBLE ? TROUBLE?

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THE RULES OF SOAP BUBBLE GEOMETRY

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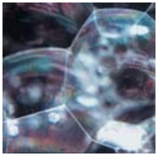


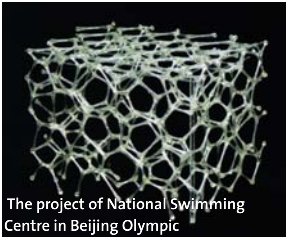
Abstracts

Let,s start with Bubbles……

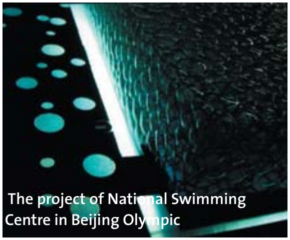
Several months ago I was amazed by the pneumatic objects and the soap bubble structure. In fact the soap bubble is one kind of pneumatic object which constrains itself to maximum volume and minimum surface area; along with the surface tension it establishes an amazing balanced and fragile structure. In this thesis I would like to explore the bubble structure, seek a possibility to use computer programming to reinterpret soap bubble structure. Architecture today is not only built by the concrete, steel and glasses anymore. Our power of the new tools will help us to discover the variety of prototypes. Besides a data-driven structure can change shape and define itself by the users. Wherefore structures will be able to adapt themselves physically to changing circumstances, instead of collecting sudden circumstances to enhance the architecture itself. The project of National swimming centre in Beijing Olympic has already revealed to use the bubble structure as roof structure to create the presentation of water. This proved that with computer programmers have introduced the method to indicate the possibilities and restrictions of structural Architecture projects.

CONCEPT

The concept is to divide research into two parts, first is to collecting the existing information of the rules of soap bubble geometry, in order to comprehend the behaviors of soap bubbles. The second part will take dodecahedron as single and simplified soap bubble unit, by using the mathematic calculation of Platonic solid, multiple the unit and create the array for the composition for soap bubbles' coordinate system. Polyhedral variations are an additional extension to contribute the production of Platonic solid coordinate system. The Surface Evolver which is the wrapping step to generate the surface of soap bubble. The goal is to generate a useful data system which has possibility to interpret the soap bubble behaviors.



The project of National Swimming Centre in Beijing Olympic



The project of National Swimming Centre in Beijing Olympic

The work of Takashi Murakami

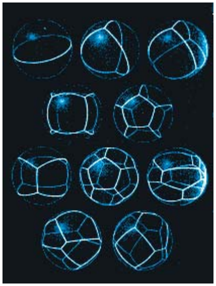
When Bubbles Meet Bubbles……

HISTORY OF SOAP BUBBLE GEOMETRY

The Introduction

The dodecahedron has 20 vertices, 30 edges and 12 faces- each with five sides. But what solid has 22.9 vertices, 34.14 edges and 13.39 faces -each with 5.103 sides? Some kind of elaborate fractal, perhaps? No, this solid is an ordinary, familiar shape, one that you can probably find in your own home. Look out for it when you drink a glass of cola or beer, take a shower or wash the dishes. Bubbles have fascinated people ever since the invention of soap. But the mathematics of bubbles and foam only really got going in the 1830s, when Belgian physicist Joseph A. Plateau began dipping wire frames into soap solution and was astounded by the results. Despite 170 years of research, we still have not arrived at complete mathematical explanations - or even descriptions of - several interesting phenomena that Plateau had observed. Soap bubbles and films are examples of an immensely important mathematical idea called a minimal surface. This is a surface whose area is the smallest possible, subject to certain additional constraints. Minimal surfaces relate to bubbles because the energy caused by surface tension in a soap film is proportional to its area. Nature likes to minimize energy-so bubbles minimize area. For example, the surface of smallest area that encloses a given volume is a sphere, and that's why isolated soap bubbles are spherical. A soap film is so thin-about a millionth of a meter-that it closely resembles an infinitely thin mathematical surface. Without some constraint, the area of a minimal surface would be zero. The most common constraints are that the surface should enclose some given volume or that its boundary should lie on some given surface or curve, or both. A bubble that forms against a flat tabletop, for example, is usually a hemisphere, and this is the smallest area surface that encloses a given volume and has a boundary lying in a plane .

Plateau's observation about the 1200 angle was quickly established as a mathematical fact. The proof is often credited to the great geometer Jacob Steiner in 1837, but Steiner was beaten to the punch by Evangelista Torricelli and Francesco B. Cavalieri around 1640. All these mathematicians actually studied an analogous problem for triangles. Given a triangle and a point inside it, draw the three lines from that point to the triangle's vertices and add up their lengths. Which point makes this total distance smallest? Answer: the point that makes the three lines meet at angles of 1200 . (Provided no angle of the triangle exceeds 1200 , that is - otherwise the desired point is the corresponding vertex.) The problem for soap films can be reduced to that for triangles by intersecting the films with a suitable plane. In 1976 Frederick J. Almgren, Jr., then at Princeton University and Jean E. Taylor; then at the Massachusetts Institute of Technology, proved Plateau's second rule about 1090 28' angles. By the spherical analogue of the Torricelli-Cavalieri theorem, these arcs must always meet in threes at angles of 1200The 1200 rule leads to a beautiful property of two coalescing bubbles. It has long been assumed on empirical grounds that when two bubbles stick together; they form three spherical surfaces, arranged as in the illustration on the opposite page. This is the Double Bubble Conjecture. If it is true, the radii of the spherical surfaces must satisfy a simple relationship. Let the radii of the two bubbles be R and S and let the radius of the surface along which they meet



Plateau's Rule for the angle between four bubble edges was proved by considering the possible ways in which six faces meet. The vertices are enclosed in a sphere, on which the faces meet at angles of 120 degrees. As shown, only 10 shapes meet this criterion; of these, only the first , three are physically plausible, because they correspond to minimal areas.

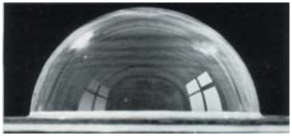
Radii of two coalescing bubbles and their common surface obey a simple relationship.

be T.

Then the relationship is : $1/R = 1/S + 1/T$.

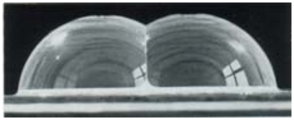
This fact is proved in Cyril Isenberg's delightful hook The Science of Soap Films and Soap Bubbles, using no more than elementary geometry and the 1200 property. All that remains is to prove that the surfaces are parts of spheres, and it is this that Hass and Schlafly achieved in 1995-but only by making the additional assumption that the bubbles are of equal volume. Their proof required the assistance of a computer; which had to work out 200,260 integrals associated with competing possibilities-a task that took the machine a mere 20 minutes! One curious fact that is known about the unequal volume case is that whatever the double-bubble minimal configuration is, it must he a surface of revolution.

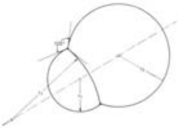
THE BUBBLE DYNAMIC

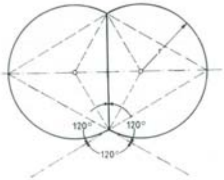


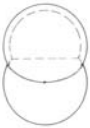
Physicists have been studying foams for a long time to investigate their dynamic properties. Although there is a great deal of published research in soft condensed matter physics on many aspects of foams, foam dynamics is still far from completely understood. Due to the instability of liquid foam, many foam experiments tend to be very difficult to perform. This is one reason why physicists developed and used computer simulations of foam structure and dynamics. Most of the approaches restrict themselves to the simulation of two dimensional foams. This reduction in dimensionality significantly decreases the complexity of the problem and most insights gained from these 2D simulations can be transferred to the 3D case.

Liquid foams are two phase systems consisting of a liquid enclosing bubbles of gas. Depending on the liquid content, the structure of the foam can vary greatly. In an extremely wet foam, the gas bubbles are spherical and separated by large amounts of liquid. At the other extreme, a very dry foam consists of extremely thin films of liquid separating the gas bubbles. Foams encountered in everyday life such as beer foam or dish washing foam are quite dry, featuring liquid films with thickness ranging from a few to several tens of microns. Where the films meet, small tubes with a triangular cross-section are formed. Known as Plateau borders, these tubes are where most of the water in a foam is contained. They are responsible for some of the characteristic visual properties of liquid foams. The surface tension in liquids causes contracting forces along the surface of the





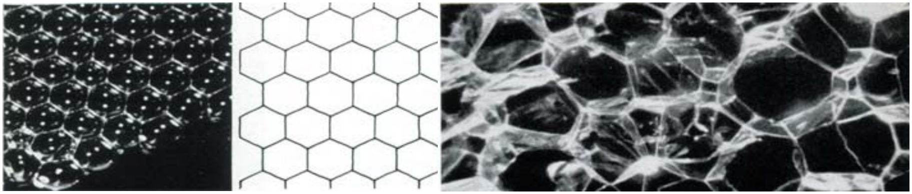




liquid. As a result, liquids always try to minimize their surface area. The reason why a bubble remains stable and does not collapse into a drop of liquid is that there is an excess pressure inside the bubble. This produces forces acting on the bubble surface that exactly cancel the forces owing to the surface tension.

The Laplace-Young Law

Laplace and Young derived an equation relating the radius of curvature R of a liquid film with surface tension to the pressure difference between the gas cells it separates: This law can be used to compute the radius of individual soap bubbles, as well as the radius of curvature of films separating two gas cells. According to this law, such a separating liquid film will be curved toward the cell with the smaller pressure. It also follows from this law that smaller bubbles have higher pressure than larger bubbles and thus the separating film between two different size bubbles will be curved towards the larger bubble.



Law of Plateau

In the 19th century, Plateau performed experiments with soap films and frameworks. He experimentally established rules about the geometric properties of soap films that were later theoretically proven. These rules state that the liquid films in a foam always meet in groups of three. At these junctions (the Plateau borders) the films always form angles of 120. The Plateau borders themselves meet in groups of four in the tetrahedral angle of 109.5 .

By inspection, a froth of soap bubbles suggests that an infinite variety of configurations can be formed by joining soap bubbles, when actually they come together in only two ways. The possible configurations are governed by a few elementary rules that have been known for more than a century. More recently, Frederick J. Almgren, Jr., and Jean E. Taylor showed that three basic rules govern the geometry of soap bubbles and that these rules are the mathematical consequence of a simple Area-minimizing Principle.
The three basic rules are:

.

1. a compound soap bubble consists of flat or smoothly curved surfaces smoothly joined together.

2. the surfaces meet in only two ways: Either exactly three surfaces meet along a

smooth curve or six surfaces (together with four curves) meet at a vertex.

3. when surfaces meet along curves or when curves and surfaces meet at points, they do so at equal angles. In particular, when three surfaces meet along a curve, they do so at angles of 120° with respect to one another, and when four curves meet at a point, they do so at angles of 109.47° (109°28’16”).

Dynamic Effects in Liquid Foams

The structure of foam changes over time for several reasons. Gravity exerts forces on the liquid, resulting in a drainage of the foam. Thus, the films get thinner over time, and the probability of film rupture increases. As more and more films break, the foam gets coarser and finally disintegrates. Additionally, in the case of froth on a liquid, new bubbles could rise from the liquid and add to the foam from below.

And ...Now... the Troubles

COMPOSITION OF DODECAHEDRON

Dodecahedron

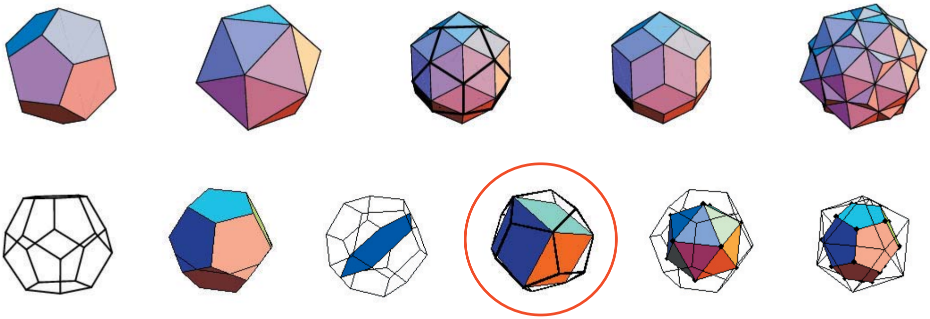
Type	Platonic
Face polygon	Pentagon
Faces	12
Edges	30
Vertices	20
Faces per vertex	3
Vertices per face	5

$$\text{Area} = 3\sqrt{25 + 10\sqrt{5}}a^2$$

$$\text{Volume} = \frac{1}{4}(15 + 7\sqrt{5})a^3$$

A dodecahedron is a Platonic solid composed of twelve pentagonal faces, with three meeting at each vertex. It has twenty vertices and thirty edges. Its dual polyhedron is the icosahedron. Canonical coordinates for the vertices of a dodecahedron centered at the origin are {(0,±1/φ,±φ), (±1/φ,±φ,0), (±φ,0,±1/φ), (±1,±1,±1)}, where φ = (1+√5)/2 is the golden mean. Five cubes can be made from these, with their edges as diagonals of the dodecahedron's faces, and together these comprise the regular polyhedral compound of five cubes. The stellations of the dodecahedron make up three of the four Kepler-Poinsot solids. The face angle of a dodecahedron is approximately 116.565 degrees. The term dodecahedron is also used for other polyhedra with twelve faces, most notably the rhombic dodecahedron which is dual to the cuboctahedron and occurs in nature as a crystal form. The normal dodecahedron is sometimes called the pentagonal dodecahedron to distinguish it. The 20 vertices and 30 edges of a dodecahedron form the basic map for a computer game called Hunt The Wumpus. Especially in roleplaying, this solid is known as a d12, one of the more common Polyhedral dice.

Caculation OF Dodecahedron



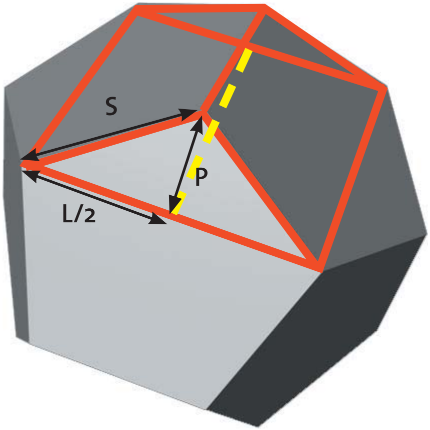
do·dec·a·he·dron

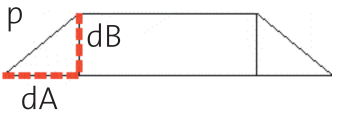
n. pl. do·dec·a·he·drons or do·dec·a·he·dra (-dr)
A polyhedron with 12 faces

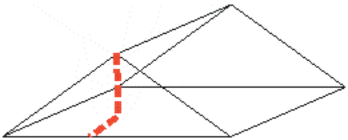
Note: The regular dodecahedron is bounded by twelve equal and regular pentagons; the pyritohedron (see Pyritohedron) is related to it; the rhombic dodecahedron is bounded by twelve equal rhombic faces.

Platonic solid

n : any one of five solids whose faces are congruent regular polygons and whose polyhedral angles are all congruent





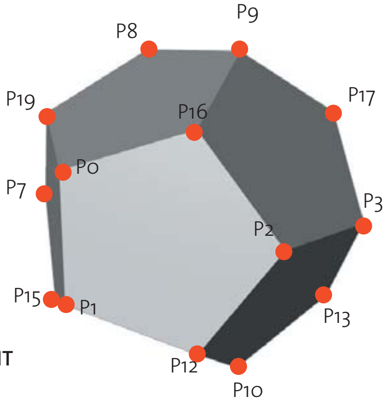


L/2=S*COS36°
P=S*SIN36°
dA=(2*S*COS36°-S)/2
dB*dB=(P*P) - (dA*dA)

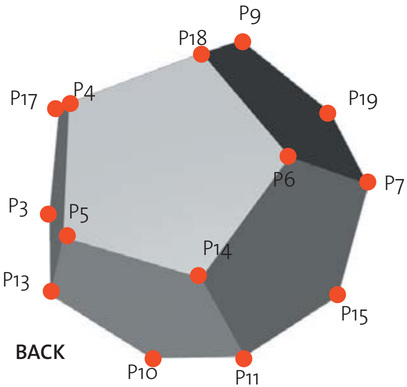
The Dodecahedron has been a source of metaphysical interest for at least 2000 years. Like a crystal or gem, its facets and symmetries compel our eyes and hearts to observe life more deeply. Some have believed that the Dodecahedron represents an idealized form of Divine thought, will, or idea. To contemplate this symbol was to engage in meditation upon the Divine. Today many people believe there is a lost knowledge residing in the past, slowly being rediscovered. It is known that figures like Pythagoras, Kepler, and Leonardo, among many, were educated in these Sacred Geometries, and held many beliefs about them and their role in the Universe.

Here we use an in internal cube to divide dodecahedron for the convenience of calculations. As the graphic on the left, we can see how the calculation and coordinate system appeared.

The Coordinates



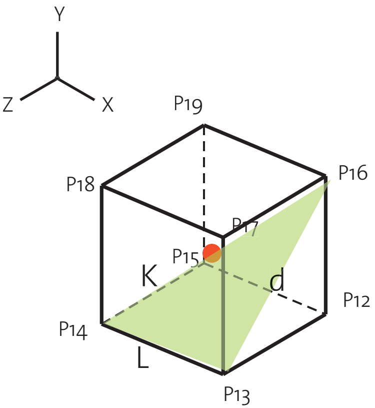
FRONT



BACK

RADIUS:100, CENTER POINT(0,0,0)

POINT	X	Y	Z	POINT	X	Y	Z
P0	0	35.682	-93.417	P10	35.682	-93.417	0
P1	0	-35.682	-93.417	P11	-35.682	-93.417	0
P2	93.417	0	-35.682	P12	57.735	-57.735	-57.735
P3	93.417	0	35.682	P13	57.735	-57.735	57.735
P4	0	35.682	93.417	P14	-57.735	-57.735	57.735
P5	0	-35.682	93.417	P15	-57.735	-57.735	-57.735
P6	-93.417	0	35.682	P16	57.735	57.735	-57.735
P7	-93.417	0	-35.682	P17	57.735	57.735	57.735
P8	-35.682	93.417	0	P18	-57.735	57.735	57.735
P9	35.682	93.417	0	P19	-57.735	57.735	-57.735



K=2*Radius
d*d=2*L*L
K*K=d*d+L*L
K*K=3*L*L
L=(√4/3)*R

L/2=57.735
dB=(L/2)-dA=22.053
dA=(2*COS36°-1)*(L/2)
=(0.618033988)*(L/2)
=35.682

POINT	X	Y	Z	POINT	X	Y	Z
P0	0	35.682	-93.417	P10	35.682	-93.417	0
P1	0	-35.682	-93.417	P11	-35.682	-93.417	0
P2	93.417	0	-35.682	P12	(L/2)	-(L/2)	-(L/2)
P3	93.417	0	35.682	P13	(L/2)	-(L/2)	(L/2)
P4	0	35.682	93.417	P14	-(L/2)	-(L/2)	(L/2)
P5	0	-35.682	93.417	P15	-(L/2)	-(L/2)	-(L/2)
P6	-93.417	0	35.682	P16	(L/2)	(L/2)	-(L/2)
P7	-93.417	0	-35.682	P17	(L/2)	(L/2)	(L/2)
P8	-35.682	93.417	0	P18	-(L/2)	(L/2)	(L/2)
P9	35.682	93.417	0	P19	-(L/2)	(L/2)	-(L/2)

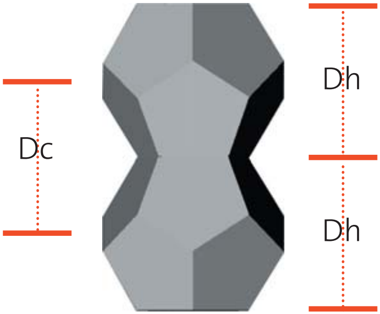
L/2=57.735
COS36°=0.809016994

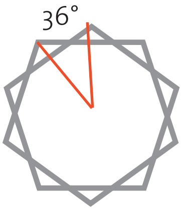
dB=(L/2)-dA=22.053
dA=(2*COS36°-1)*(L/2)
=(0.618033988)*(L/2)
=35.682

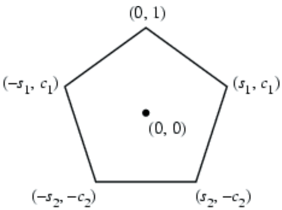
POINT	Coordinate	dX	dY	dZ
P0	M.16.19	0	-dB	-dA
P1	M.12.15	0	dB	-dA
P2	M.12.16	dA	0	dB
P3	M.13.17	dA	0	-dB
P4	M.17.18	0	-dB	dA
P5	M.13.14	0	dB	dA
P6	M.14.18	-dA	0	-dB
P7	M.15.19	-dA	0	dB
P8	M.18.19	dA	dB	0
P9	M.16.17	-dA	dB	0
P10	M.12.13	-dB	-dA	0
P11	M.14.15	dB	-dA	0

The Rules of Muti-Units Array

Faces	POINT
F0	0,16,2,12,1
F1	2,16,9,17,3
F2	2,3,13,10,12
F3	0,1,15,7,19
F4	0,16,9,8,19
F5	11,15,1,12,10
F6	9,19,7,6,18
F7	6,7,15,11,14
F8	5,14,11,10,13
F9	3,13,5,4,17
F10	4,18,6,14,5
F11	8,17,4,18,9



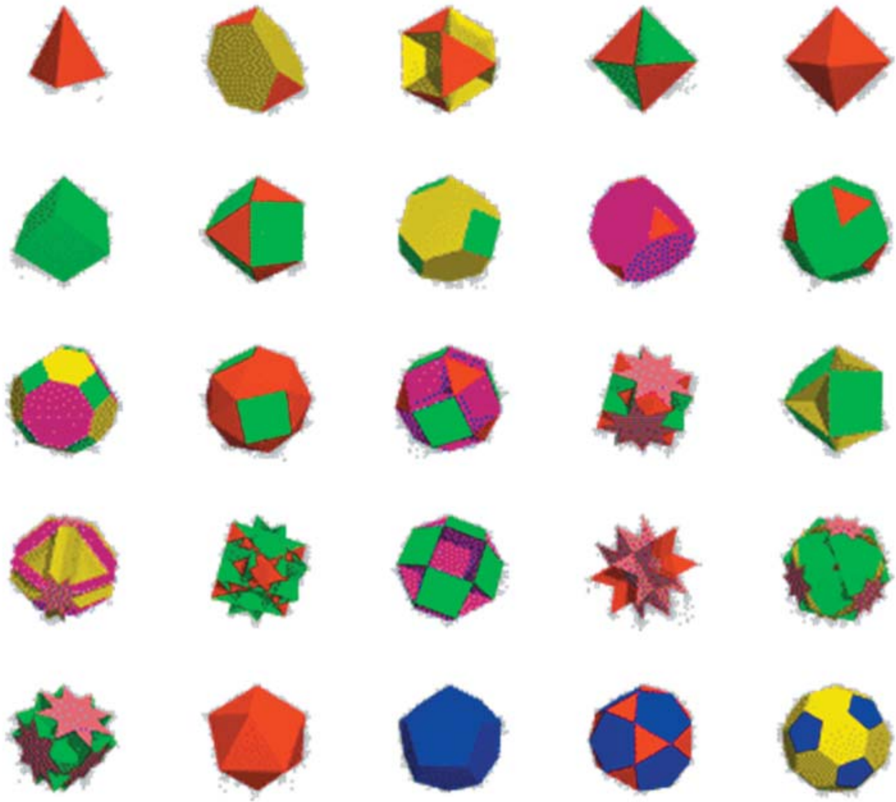


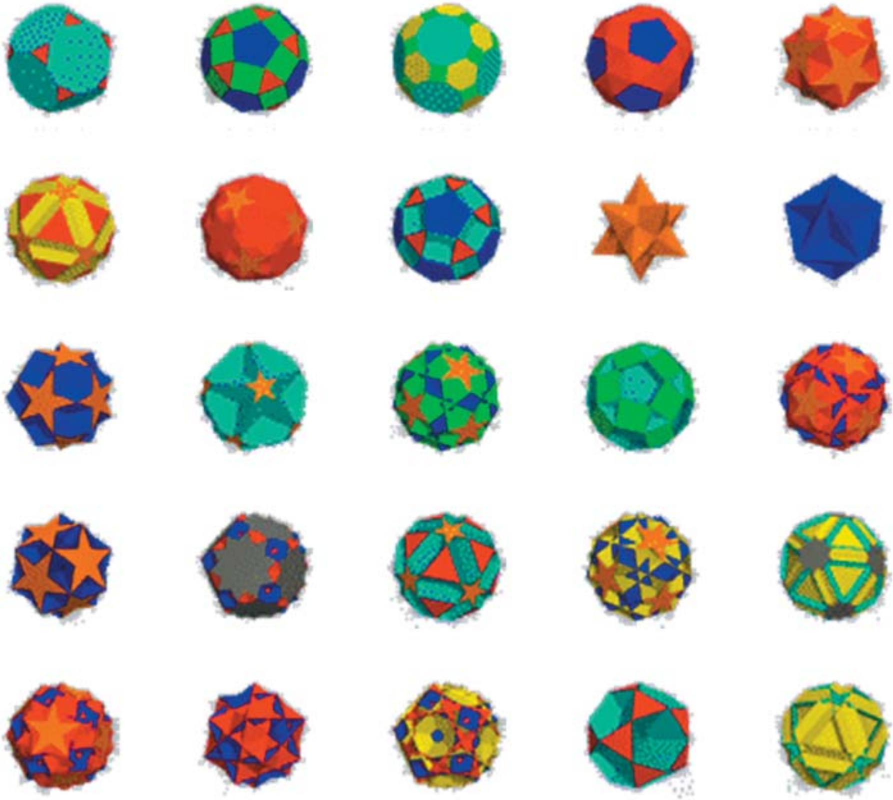


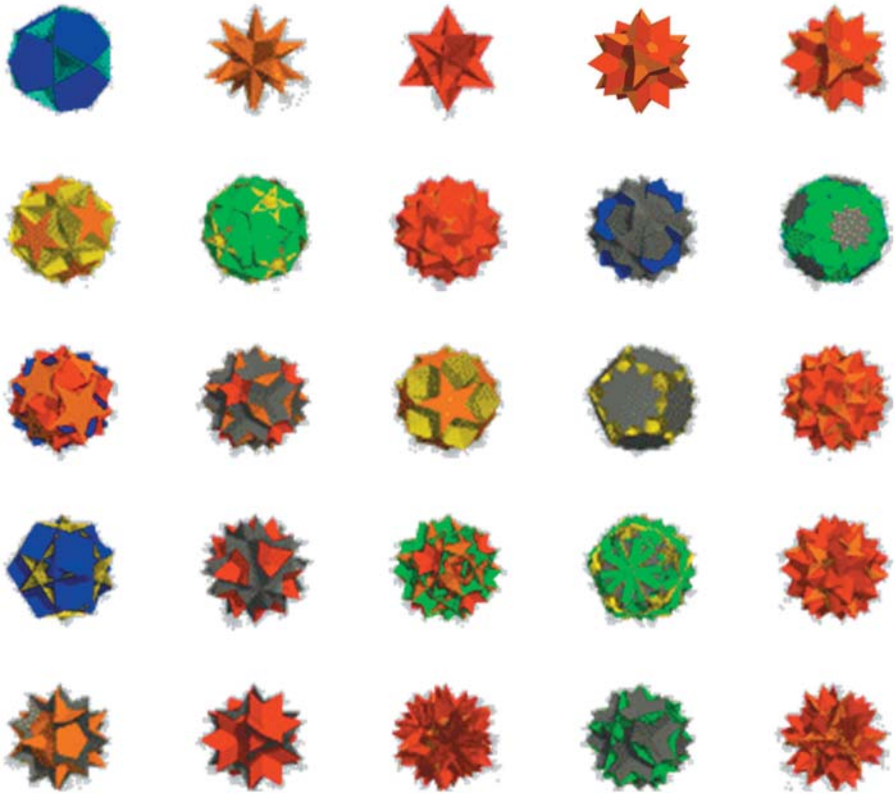
$$c_1 = \cos\left(\frac{2\pi}{5}\right) = \tfrac{1}{4}(\sqrt{5}-1) \qquad s_1 = \sin\left(\frac{2\pi}{5}\right) = \tfrac{1}{4}\sqrt{10+2\sqrt{5}}$$

$$c_2 = \cos\left(\frac{\pi}{5}\right) = \tfrac{1}{4}(\sqrt{5}+1) \qquad s_2 = \sin\left(\frac{4\pi}{5}\right) = \tfrac{1}{4}\sqrt{10-2\sqrt{5}}.$$

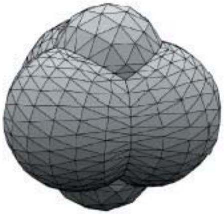
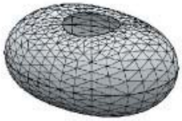
All Polyhedral Variations







The Surface Evolver

From left to right, Schwarz P surface, cluster of five bubbles, shape of a droplet on a spinning rod.

The Surface Evolver is a software developed by Ken Brakke at University of Minnesota since the early 1990s. It is specifically designed to find minimal energy configurations of interfaces under almost arbitrary constraints, and to find the evolution path towards those optimal states. The Evolver can handle interfaces of arbitrary topology and almost arbitrary dimension. Interactive modifications of the computed surface are also possible. The software has been used to study many problems in mathematics and the physical sciences, such as minimal surfaces, sphere eversion, or lipid vesicle shapes. See figures above for a few examples.

Maybe the best feature of the Surface Evolver is that it is freely available to everyone, for a variety of computer platforms. Downloads, documentation, and more information can be found in : http://www.susqu.edu/facstaff/b/brakke .

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More
Bubbles??
More
Troubles???

THE SCRIPT

The VectorScript

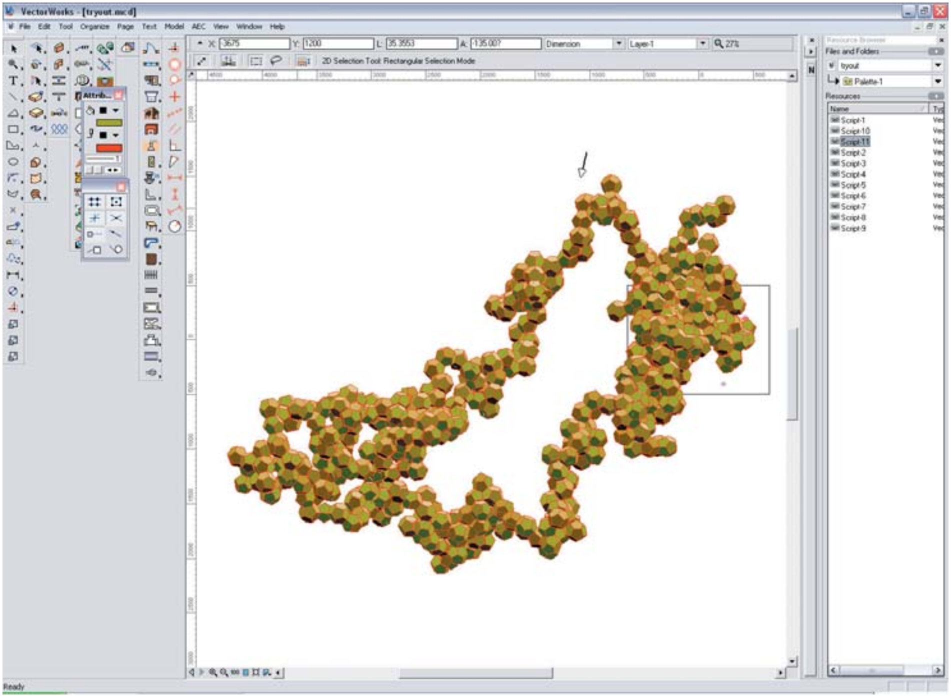
```
PROCEDURE dodecahedron;

VAR
        cCenter,cVx,cVy,cVz:VECTOR;
        p:ARRAY[0..19] OF VECTOR;
        side:ARRAY[0..12,0..5] OF INTEGER;
        sideCenter:ARRAY[1..12] OF VECTOR;
        pa,pb,pc:VECTOR;
        newSide,i:INTEGER;

PROCEDURE doDode( center,vx,vy,vz:VECTOR; Radius:REAL);
VAR
        k,i,j,L,myUnit,dA,dB,centerX,centerY,centerZ:REAL;
        h:HANDLE;
        lob:HANDLE;
        pIndex,sIndex,psIndex:INTEGER;

BEGIN
```

```
j:=Sqrt(4/3);
i:=Cos(Pi/5);
k:=((2*i)-1);
L:=j*Radius;
myUnit:=(L/2);
dA:=(k*myUnit);
dB:=(myUnit-dA);

P[12]:=center+myUnit*vx-myUnit*vy-myUnit*vz;
P[13]:=center+myUnit*vx-myUnit*vy+myUnit*vz;
P[14]:=center-myUnit*vx-myUnit*vy+myUnit*vz;
P[15]:=center-myUnit*vx-myUnit*vy-myUnit*vz;
P[16]:=center+myUnit*vx+myUnit*vy-myUnit*vz;
P[17]:=center+myUnit*vx+myUnit*vy+myUnit*vz;
P[18]:=center-myUnit*vx+myUnit*vy+myUnit*vz;
P[19]:=center-myUnit*vx+myUnit*vy-myUnit*vz;

P[0]:=(P[16]+P[19])*0.5-dB*vy-dA*vz;
P[1]:=(P[12]+P[15])*0.5+dB*vy-dA*vz;
P[2]:=(P[12]+P[16])*0.5+dA*vx+dB*vz;
P[3]:=(P[13]+P[17])*0.5+dA*vx-dB*vz;
P[4]:=(P[17]+P[18])*0.5-dB*vy+dA*vz;
P[5]:=(P[13]+P[14])*0.5+dB*vy+dA*vz;
P[6]:=(P[14]+P[18])*0.5-dA*vx-dB*vz;
P[7]:=(P[15]+P[19])*0.5-dA*vx+dB*vz;
P[8]:=(P[18]+P[19])*0.5+dB*vx+dA*vy;
P[9]:=(P[16]+P[17])*0.5-dB*vx+dA*vy;
P[10]:=(P[12]+P[13])*0.5-dB*vx-dA*vy;
```

```
P[11]:=(P[14]+P[15])*0.5+dB*vx-dA*vy;

side[1,0]:=0;side[1,1]:=16;side[1,2]:=2;side[1,3]:=12;side[1,4]:=1;   {F0: 0,16,2,12,1}
side[2,0]:=2;side[2,1]:=16;side[2,2]:=9;side[2,3]:=17;side[2,4]:=3;   {F1: 2,16,9,17,3}
side[3,0]:=2;side[3,1]:=3;side[3,2]:=13;side[3,3]:=10;side[3,4]:=12;  {F2: 2,3,13,10,12}
side[4,0]:=0;side[4,1]:=1;side[4,2]:=15;side[4,3]:=7;side[4,4]:=19;   {F3: 0,1,15,7,19}
side[5,0]:=0;side[5,1]:=16;side[5,2]:=9;side[5,3]:=8;side[5,4]:=19;   {F4: 0.16,9,17,3}
side[6,0]:=11;side[6,1]:=15;side[6,2]:=1;side[6,3]:=12;side[6,4]:=10;   {F5: 11,15,1,12,10}
side[7,0]:=8;side[7,1]:=19;side[7,2]:=7;side[7,3]:=6;side[7,4]:=18;  {F6: 8,19,7,6,18}
side[8,0]:=6;side[8,1]:=7;side[8,2]:=15;side[8,3]:=11;side[8,4]:=14; {F7: 6,7,15,11,14}
side[9,0]:=5;side[9,1]:=14;side[9,2]:=11;side[9,3]:=10;side[9,4]:=13;   {F8: 5,14,11,10,13}
side[10,0]:=3;side[10,1]:=13;side[10,2]:=5;side[10,3]:=4;side[10,4]:=17; {F9: 3,13,5,4,17}
side[11,0]:=4;side[11,1]:=18;side[11,2]:=6;side[11,3]:=14;side[11,4]:=5; {F10: 4,18,6,14,5}
side[12,0]:=9;side[12,1]:=17;side[12,2]:=4;side[12,3]:=18;side[12,4]:=8; {F11: 8,17,4,18,9}



FOR sIndex:=1 TO 12 DO BEGIN
	sideCenter[sIndex].x:=0;
	sideCenter[sIndex].y:=0;
	sideCenter[sIndex].z:=0;

	FOR pIndex:=0 TO 4 DO BEGIN

		sideCenter[sIndex]:=sideCenter[sIndex]+P[side[sIndex,pIndex]];

	END;
	sideCenter[sIndex]:=sideCenter[sIndex]*(1.0/5.0);
	sideCenter[sIndex]:=sideCenter[sIndex]-center;
END;
```

```
	FOR sIndex:=1 TO 12 DO BEGIN
		{NameObject(Concat('side',sIndex));}
		ClosePoly;
		BeginPoly3D;
		FOR pIndex:=0 TO 4 DO BEGIN

			Add3DPt(P[side[sIndex,pIndex]].x,P[side[sIndex,pIndex]].
y,P[side[sIndex,pIndex]].z);

		END;
		EndPoly3D;

	END;


END;

BEGIN
	ccenter.x:=0;
	ccenter.y:=0;
	ccenter.z:=0;
	cvx.x:=1;cvx.y:=0;cvx.z:=0;
	cvy.x:=0;cvy.y:=1;cvy.z:=0;
	cvz.x:=0;cvz.y:=0;cvz.z:=1;
	doDode(ccenter,cvx,cvy,cvz,100);

	FOR i:=1 TO 650 DO BEGIN
		newSide:=1+trunc(random*12);
```

```
        ccenter:=ccenter+2.0*(sideCenter[newSide]);
        pa:=p[side[newSide,0]];
        pb:=p[side[newSide,1]];
        pc:=(pa+pb)*0.5;
        cvz:=UnitVec(ccenter-pc);
        cvy:=UnitVec(pb-pa)*1.0;
        cvx:=UnitVec(CrossProduct(cvy,cvz));
        doDode(ccenter,cvx,cvy,cvz,100);
    END;

END;


run(dodecahedron);
```


VectorWorks - [tryout.mcd]
File Edit Tool Organize Page Text Model AEC View Window Help
2D Selection Tool: Rectangular Selection Mode
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