

FORM - RULE

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Architects do not produce geometry, they consume it. Such at least would be the inevitable conclusion of anyone reviewing the history of architectural theory."

Robin Evans

... there is geometry in architecture

Toni Kotnik

Within contemporary architectural design the form – rule relationship is often understood as the application of geometric rules in a generative process of form-finding, that is rules are a logico-algebraic text out of which architectural form emerges through the manipulation of data. By looking at the etymological roots of mathematics another reading of geometry can be uncovered that relates geometry back to bodily experience and the question of spatial orientation. This enables the re-introduction of the body into contemporary discourse of digital architecture.



[1] Robin Evans, The Projective Cast: Architecture and Its Three Geometries (MIT Press, 1995), xxvi

[2] Rivka Oxman, "Theory and design in the first digital age", Design Studies (2006), 27(3), pp. 229 – 265 or Rivka Oxman, "Digital architecture as a challenge for design pedagogy: theory, knowledge, models and medium", Design Studies (2008), 29(2), pp. 99 – 120

[3] In 1963, Ivan Sutherland's Sketchpad program demonstrated that computers could be used for drafting and modelling. By the mid-1990s, architectural practice without graphics software had become unimaginable, and today, digital design technologies have been adopted almost universally as the predominant means of production in architectural practice That is architecture is taking part in an "intellectual revolution [that] is happening all around us, but few people are remarking on it. Computational thinking is influencing research in nearly all disciplines, both in the sciences and the humanities . It is changing the way we think." Alan Bundy, "Computational Thinking is Pervasive", Journal of Scientific and Practical Computing (2007), I (2), pp. 67 - 69

[4] Toni Kotnik, "Digital Architectural Design as Exploration of Computable Functions", International Journal of Architectural Computing (2010), in press

[5] Toni Kotnik, "Das Experiment als Entwurfsmethodik Zur Möglichkeit der Integration naturwissenschaftlichen Arbeitens in die Architektur", A. Moravánszky and A.H. Kirchengast, Experiments in Architecture (Jowis 2010), in press

> [6] Morris Kline, Mathematics in Western Culture (Oxford University Press, 1966), pp. 184 – 186

[7] "Computationally advanced design techniques such as scripting and parametric modeling are becoming a pervasive reality such that it is no longer possible to compete within the contemporary avant-garde architecture scene without mastering and refining them." Schumacher, P: Parametricism – A New Global Style for Architecture and Urban Design, AD Architectural Design, 77/4 (2009), 17 I. Introduction

In his introduction to his study on the relationship of projective geometry and architecture the historian and theoretician Robin Evans pointed out that "geometry is one subject, architecture another, but there is geometry in architecture. ... Geometry is understood to be a constitutive part of architecture, indispensable to it, but not dependent on it in any way. The elements of geometry are thus conceived as comparable to bricks that make a house, which are reliable manufactured elsewhere and delivered to site ready to use. Architects don't produce geometry, they consume it [1]."

Over the past two decades, the architectural consumption of geometry has been intensified with the emergence of new methodologies in architectural design that exploit the computer as a design tool. This has generated a varied set of digital skills and a new type of architectural knowledge and has led to a re-examination of current design theories [2] due to the unavoidable changes of discipline-immanent thinking by means of computation [3]. With the integration of the computer into the design process, Evan's geometric bricks have been transformed into Turing-machines, i.e. simple abstract devices as representation of computable mathematical functions that mediate between input and output [4].

The consumption of geometry by todays digital architecture, therefore, is

characterized by a turning towards quantification of causal relationships between architectural entities and the algorithmic description of their interaction [5].

Such need for quantification of causal relationships in a digital design process mirrors the shift in the sciences from an Aristotelian thinking to a Galilean thinking in the 17th century. This paradigmatic shift was characterized the replacement of metaphysical explanations for the necessity of an observed causality by the guantitative recording of phenomena by means of measurable entities like the change in the distance between points over time ren. Galilei explained his method of guantification of nature in detail in his book Discorsi e dimostrazioni matematiche, intorno a due nuove scienze, published in 1638 despite an existing ban by the church. Completed and refined in 1687 by Isaac Newton's Philosophiae naturalis principia mathematica the Galilean method turned out to be so powerful that already one century later it was seen as unquestionable cornerstone in the scientific study of nature.

The digital in architecture, thus, has the potential to introduce scientific thinking and methodology into the design process [7]. At the same time, it causes an already ongoing shift in the understanding of the role of geometry in architecture. With the need for quantification of causality geometry transforms into a descriptive, logico-algebraic text for the emergence of architectural form out of the manipulation of data.

2. Mathemata

Such a formalistic reduction of ma-thematics, however, does not reflect the essence of mathematics. Already Husserl had pointed out that the scientific method of quantification of causal relationships is an emptying of mathematical thinking [19]. Etymologically, mathematics has its roots in the Greek ta mathemata, which means what can be learned where learning, mathesis, is about the recognition of the unchanged, the stable, of the Being in a world of constant Becoming [19].

For Heidegger "this genuine learning is an extremely peculiar taking, a taking where one who takes only takes what one basically already has. ... The mathemata, the mathematical, is that 'about' things which we already know. Therefore we do not first get it out of things, but, in a certain way, we bring it with us [10]." Mathemata, therefore, is bound to the human perception and its ability to identify reoccurring pattern.

Mathematics is the science of patterns! It is about the examination of numerical patterns, patterns of shape, patterns of motion, patterns of behavior. They can be real or imagined, visual or mental, static or dynamic, qualitative or quantitative [11]. Mathemata is the human search for patterns as a means of orientation. The world is a construct whose main reference point is the here and now of the body. It is from the body that surrounding reality is perceived, structured, and accessed [12]. In its original meaning, mathematics is about relating the body with the world around. As such, mathemata is about orientation.

The concept of orientation has its etymological origins in this subjective quotidian view of the world. The noun Orientierung (orientation) is derived from the Middle High German orient, meaning "the east," and the Latin oriens, the participle form of oriri, which means "to rise or go up." The orient as a description of the east is thus a reference to the rising sun [13]. The act of orientation is therefore always an act of positioning oneself in the environment and, as a result, of assuring oneself of one's own existence through alignment with the east; an act of positioning oneself in relation to the rising sun.

Mathemata as the search for orientation is therefore always a manifestation of being in the world, of integrating oneself, of creating borders, and giving oneself space. As an act of giving oneself space, orientation refers to the original medieval conception of space as rum, a cleared settlement site. According to Heidegger, space is that which is intrinsically marked out and placed within borders [14]. Orientation can therefore be understood [8] Edmund Husserl, Die Krisis der europäischen Wissenschaften und die transzendentale Phänomenologie. Eine Einleitung in die phänomenologische Philosophie (1936)

[9] Platon, Timaios

[10] Martin Heidegger, "Modern Sciences, Metaphysics, and Mathematics", David F. Krell, Martin Heidegger: Basic Writing (Harper Collins, 1992), pp 271 – 305

[11] Keith Devlin, Mathematics: The Science of Patterns (Scientific American Library, 1994)

[12] Peter Zec, Orientierung im Raum (Mabeg, 2002), pp. 17 – 19.

[13] Many languages make this conceptual connection between the east and the rising sun. Cf. Cecil Brown, "Where do Cardinal Direction Terms Come From?" Anthropological Linguistics 25 (1983): pp. 121 – 61; or Bernd Heine, Cognitive Foundations of Grammar (Oxford University Press, 1997).



as the active and constructive act of appropriating the environment by setting boundaries.

3. Geometrein

In ancient Egypt, this setting of boundaries as a method of appropriating the environment was the main job of the geometer (literally: "measurer of the earth"). Hero of Alexandria (c. 20-62 AD) writes: "As the old tradition teaches us, the earliest form of geometry dealt with measuring and dividing up estates, which is why it was called 'land measuring.' It was the flooding of the Nile that gave the Egyptians the idea of making measurements, because many pieces of property that were exposed at the edge of the river disappeared as the water rose and only reappeared after it had sunk. It was not always possible to determine who owned them rist."

One of the oldest methods of orientation was this form of geometrization the measurement of land by geometers called Harpedonaptae, or "rope stretchers" [16]. Geometrein, the activity of measuring the land, can therefore be understood as a means of orienting oneself by creating space, as a carving out of physical space by setting boundaries. This was made particularly clear by the ancient Egyptian foundation-laying ceremony, the "stretching of the rope," [17] which was the first practical act performed in temple construction. The sacred area was marked out by ropes and stakes, and the building's orientation and dimensions were set.

4. Euclidean geometry

The Egyptians' practical surveying skills were legendary and won the admiration of the Greeks. Pre-Socratic thinkers such as Thales of Milet (624-546 BC) and Pythagoras of Samos (c. 572-490 BC) traveled through Egypt to become acquainted with the skills and knowledge of Egyptian geometers. This knowledge, which was based on a set of constructs and special model cases, marked the start of the scientification of geometry and is reflected in the Elements, a systematic portrayal of the geometric knowledge of antiquity by Euclid of Alexandra (c. 365-300 BC) [18].

Euclid traces all mathematical pro-positions back to five basic axioms that are known as the Five Postulates of Euclidean geometry: I.A straight line may be drawn from any point to another. 2. A straight line segment may be extended indefinitely along a straight line. 3. A circle may be described with any point as its center and any distance as its radius, 4, All right angles are identical, 5, Finally, if two straight lines intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two straight lines must at some point intersect on that same side if extended indefinitely,

The validity of the propositions in the Elements rests on the validity of these postulates. According to Euclid, this validity is self-evident and requires no further explanation. We can therefore assume that the postulates represent abstract descriptions of daily experience, including the daily experience of measuring land. After all, the first three postulates can be seen as directly formalizing the fundamental operations of the Harpedonaptae, the rope-stretching land measurers. The origins of the fourth postulate can also be understood in a similar way. The Egyptian geometers used both a rope divided into equal segments and a Pythagorean triangle to measure a right angle This rope method enabled them to compare angles, even those that were non-adjacent [20]. Along with the formation of straight lines and circles, the construction of triangles was one of the most important tasks in land surveying, and it is described in the fifth postulate. This explains how to determine the intersection point of two lines by extending them indefinitely – a method that can be compared to the sighting of points and the measurement of geometric shapes using a dioptra, an ancient surveying instrument that Euclid mentions in his works

In view of these parallels between practical experience and theoretical abstraction, we can assume that, for Euclid, the evident nature of the geometric postu-

on astronomy [21].

lates resulted from geometrein, that is, from the constructive act of measuring land. By means of logical connection and building on the direct experience and empirical verifiability of geometry in everyday use - new statements are possible that can doubtless be interpreted as statements about physical reality. One can therefore say that in Euclid's work, logical thought as a method of derivation does not transcend the direct experience of the environment but is congruent with it. Consequently, Euclid's Elements exemplifies a form of ancient thought that is characterized by the assumption of a fundamental and absolute agreement between subject matter and its nominal determination. Conceptual understanding is equivalent to physical understanding. Euclidean geometry is geometrein, method of measuring land by means of logical thought; an act of physical orientation based on the creation of an intellectual structure.

5. Metrics

Nevertheless, the knowledge and skills involved in land measurement were not developed to enable people to orient themselves by creating geometrically structured spaces. They resulted from the Egyptian taxation system, which imposed taxes based on the size of the farmed land. It is precisely this quantitative determination of size that is impossible in Euclidean geometry, since its postulates merely allow us to understand – in [19] Corinna Rossi, Architecture and Mathematics in Ancient Egypt, (Cambridge University Press, 2004), pp. 157 – 59.

[20] Euclid does not base his definition of a right angle on the number of degrees (90), bur on the intersection of two straight lines. If two adjacent angles are equally large at the point of intersection, the intersecting angle is described as a right angle. This means that a practical comparison of angles at different points of intersection requires a theory of measurement – the "rope triangle" – which is theoretically expressed by Postulate 4.

[21] Michael J.T. Lewis, Surveying Instruments of Greece and Rome (Cambridge University Press, 2001).

[14] Martin Heidegger; "Bauen Wohnen Denken,"Vorträge und

[15] Hero of Alexandria, "Metrica,"

Heronis Alexandrinus opera quae

Harpedonapten oder Seilspanner

Physik, vol. 1 (1931), pp. 256 - 57

[17] Corinna Rossi, Architecture

(Cambridge University Press,

2004), pp. 148 – 53.

University Press, 1884),

pp. 132 – 33.

and Mathematics in Ancient Egypt,

[18] James Gow, A Short History

of Greek Mathematics (Cambridge

und Seilknüpfer," Quellen und

Studien zur Geschichte der

Mathematik, Asrronomie und

supersunt omnia, ed. Wilhelm

[16] Solomon Gandz, "Die

Aufsätze (Neske, 1967)

pp. 139 - 56.1

Schmidt, 1976



[22] The primacy of the numerical relations between whole numbers in Greek thinking is characteristic of this understanding. They run through Plato's work as musical harmonies and reappear in the manner in which Vitruvius describes temple structures using numerical ratios and pillar diameters as referential values. For a comprehensive discussion of what are known as commensurable numbers, see Paul Naredi-Rainer, Architektur und Harmonie: Zahl, Mass und Proportion in der abendländischen Baukunst (DuMont, 1982).

[23] It was not until the seventeenth century that Euclidean geometry was connected to metrics by the coordinate system introduced by Rene Descartes and Pierre de Fermat This coordinate system plays a central role in Descartes' philosophy since it shows the dualism of the res extensae of the extended world of objects and the res cogitans of the unextended world of the mind.To a certain degree, one may understand dualism as an examination of the question of orientation within space

[24] Peter Zec, Orientierung im Raum (Mabeg, 2002), 25.

[25] Peter Berger and Thomas Luckmann, Die gesellschaftliche Konsrruktion der Wirklichkeit : Eine Theorie der Wissensoziologie (Fischer, 1997).

[26] Erhard Scholz, "Riemann's Vision of a New Approach to Geometry," Lecture Notes in Physics, vol. 402 (1992).

[27] Johann Friedrich Herbart, Über die Moglichkeit und Nothwendigkeit, Mathematik auf Psychologie anzuwenden, 1824. a step-by-step fashion – how geometric figures are formed without making any statement about dimensions. Constructs of Euclidean geometry are therefore scale-independent and merely describe the relations between different geometric elements [22].

Only when geometry is contextualized does it become a real orientation aid in physical reality, and only when geometric constructs are linked to the experienced human scale does Euclidean geometry become geometrein. The introduction of metrics, that is to say, a system for defining spatial dimensions, made it possible to transpose geometry into space and allowed people to orient themselves by means of the structure of the spatial environment, with their position as the central reference point [23].

The limits of Euclidean geometry illustrate that the act of orientation, as an active measurement of surrounding space, determines a structure that is both geo-metric and metric in nature. Both structures can be understood as cognitive structures of knowledge generated by perception and experience – structures that, as individual patterns of action, shape the individual perception of the world, the world view or weltanschauung, and, conversely, that impinge on perception [24]. Spatial orientation, which results from this subjective world view, is therefore also subjective. A shared world view emerges only when individual world views are compared, since as social beings humans always orient themselves to fellow human beings and their orientation behavior. Socially accepted orientation behavior is therefore based on a social construct of reality **r251**.

6. Riemann Manifolds

This comparison of individual structures formed the conceptual foundation for the advancements in geometry made in the work of Bernhard Riemann (1826-1866). The concept of the "manifold" described in his 1854 habilitation lecture "Über die Hypothesen, welche der Geometrie zu Grunde liegen" ("On the Hypotheses Which Lie at the Basis of Geometry"), is based primarily on the work of Johann Friedrich Herbart (1776-1841), who succeeded Kant as professor in Konigsberg [26]. According to Herbart, as people move through space, they have a variety of perceptions that do not have an immediate effect on consciousness. Herbart argues that these perceptions undergo a "graded fusion" in the mind [27]. This process of fusion, which Herbart describes as having serial form, "glues" together the individual perceptions to form a holistic geometric image and cognitive knowledge-structure without abrogating the individual perception.

Riemann's idea of manifolds renders Herbart's psychological concepts more concrete and precise for mathematical

application. A manifold consists of a large number of local maps - that is, descriptions of a limited environment based on Euclid's ideas about space. The sum of all these local manifold maps describes the sum of cartographically defined points and is called an atlas. Furthermore, for the maps of contiguous spatial regions there exists a transformation rule that clarifies transitions from one map to another by adapting local metrics. In this operation the local maps become increasingly distorted along their edge, which means that dimensions are continually changed in order to facilitate a seamless, smooth transition between maps.

Riemann's concept of manifolds universalizes a form that we are familiar with from cartography: atlases as collections of maps and abstract geometric images of the earth. Maps of contiguous regions must be successively overlaid, twisted, and distorted in order to piece together a collection of images that form a comprehensive image of the earth (the globe). The manifold is therefore a mathematical construct that is meant to facilitate the study of the interaction between local and global geometry by rendering metric structures more flexible.

7. Computability

Euclid's geometry and its universalized form in Riemann's manifolds are examples of how mathematical concepts can be created by formalizing the act of orientation. With the integration of computers into the design process, we can observe again a fundamental shift in the conception of geometry, a shift visible in the emergence of computational geometry **r281**.

This conceptual shift is caused by the fact that computers are actively shaping the way we as users approach design questions. It was Merleau-Ponty who pointed out that as humans have to see our bodies not only as the physical context or milieu of cognitive mechanisms, but also as a living, experiential structure that is both biological and phenomenological [29]. Human understanding of the world, therefore, depends in large part on the interaction of the body with its environment [30]. Every tool mediates this interaction because of its specific usage, thereby influencing the perception of the user and his way of thinking. The paradigmatic changes in many scientific disciplines demonstrate that this is true in particular for the computer despite the seemingly unspecific neutrality of the machine that enables its versatile application.

Using a computer always means, without exception, the necessary limitation on computable functions as mediator between input and output [31]. The purpose in the integration of computers as tools into the design process lies in the conscious exploration of the potential of the defining elements of a computable function as design tools: of the formal

[28] Computational approaches to geometry can be found already in antiquity. As a new discipline in computing science, however, modern computational geometry has its origin in the late 1960s and early 1970s in the Ph.D. thesis of Michael Shamos at Yale University in 1978 and of Robin Forrest at Cambridge University in 1968 as well as the formal investigation into geometric properties of figures conducted by Marvin Minsky and Seymour Papert as part of their research into artificial intelligence.

[29] Francisco Javier Varela, The Embodied Mind: Cognitive Science and Human Experience (MIT Press, 1991)

[30] Shaun Gallagher, How the Body Shapes the Mind (Oxford University Press, 2006)

[31] Toni Kotnik, "Digital Architectural Design as Exploration of Computable Functions", International Journal of Architectural Computing, vol. 8, no. 1 (2010), pp. 1 – 16



relationship between sets of entities, of quantifiable properties of these sets of entities, and of algorithmic transformations and interaction of different quantifiable properties. Digital design, thus, is not about the formalization of design processes or the automatization of decision making like for example in William Mitchell's The Logic of Architecture [32] but about the interaction of formal processes with architectural thinking; it's not about computerization but much more about computation [33].

As Kostas Terzidis puts it: "Computation is about the exploration of indeterminate, vague, unclear, and often ill-defined processes; because of its exploratory nature, computation aims at emulating or extending the human intellect. It is about rationalisation, reasoning, logic, algorithm, deduction, induction, extrapolation, exploration, and estimation," When Alan Turing formalized the idea of computation in the 1930s he had in mind a description of the process of calculation by human beings or as Wittgenstein described it: "Turing's 'Machines'. These machines are human who calculate [34]." This means, computational thinking has to be seen as the logical exploration of the space of the possible, as rational measuring of the imaginable.

8. Conclusion

Looking at the development throughout history we can conclude, that the abstract nature of geometry is not so much the expression of pure logic as it is an attempt to make human perception and experience more conceptually precise and transform them into geometrein in the sense of creating orientation in space through the act of measurement. The different resulting spatial ideas are the product of the different metrics of measurement: starting point of Euclidean geometry is the abstract technique of measuring the physical environment, Riemann's manifolds are based on the cartographic measurement of an internal cosmos of the imagination, and Computational Geometry is based on the procedural measurement of the thinkable . From the haptic geometry of antiquity, to the visual geometry of modernity, and the imaginable geometry of contemporary times we can see formalizations of various means of measurement of the space of human existence.

In general, the act of measuring is always a political act, a reorientation within political space. This is made especially apparent by developments in the eighteenth century, when measurement became a central perceptual category in academic and political discourse. The impressive precision and self-evident nature of numbers made them a seemingly objective yardstick for political action and the favored medium for political communication [35]. The abstract nature of the concept of measurement in Riemann's work, together with the observed shift of physical space into

perceptual space, opened the way for an extension of the concept of space and the application of spatial thought to social and cultural phenomena [36].

It therefore cannot surprise that the concept of Riemann's manifold and the rhizome as its discrete version is of crucial importance for Gilles Deleuze and Felix Guattari in their study of contemporary culture summarized in A Thousand Plateus: Capitalism and Schizophrenia [37]. Through a layering of seemingly unrelated subjects a "graded fusion" is provoked out of which a holistic but individual picture of society, a so-called body without organs, emerges, Not unlike Heidegger's mathemata this results in a world that "begins to signify before anyone knows what it signifies; the signified is given without being known [38]."

Inspired by this thinking, Greg Lynn has been trying to argue for a new kind of understanding of geometry in architecture; one that is no longer tied to a transcendental value system, but is an adequate expression of the contemporary reality [39]. His valid critic of the traditional use of geometry as symbolic language, however, has not led to a rethinking of the relationship between body and geometry but rather to a replacement of systems of proportions as guidelines for classical geometric constructions by a system of parameters and weights as guidelines for constructions in NURBS- geometry [40]. That is, Lynn has merely replaced one system of quantification by another system of quantification and to some extend reconstructed the Galilean method of quantification in the realm of digital architecture. This has strongly influenced our contemporary understanding of geometry and the incorporation of environmental context into the design process by means of measurable physical entities like wind pressure, sun angle or heat differences.

Measuring space is a central activity for structuring our surroundings and an important orientation element in the human environment. However, this environment must not be understood in a limited way as the purely physical environment. It must be seen more broadly and comprehensively as a multilayered perceptual and experiential space. This expanded understanding of space and the incorporation of the subject into space transforms measurement into an act of individual orientation. The importance of such bodily measuring of space, of geometrein, is not only justified by philosophers like Heidegger, Merleau-Ponty or Deleuze but also by developments in contemporary neuroscience. What emerges is an understanding of architecture as a primarily emotional and perceptual experience grounded in biological values that have to be put forward in the design process [41]. What is needed is a new mathesis of architectural geometry.

[36] This rediscovery of spatiality in the social and cultural sciences is often described as the "spatial turn." See, for example, Doris Bachmann-Medick, Cultural Turns: Neuorientierung in den Kulturwissenschaiten (Rowohlt, 2006); Karl Schlagel, Im Raum lesen wir die Zeit: Über Zivilisationsgeschichte und Geopolitik (Hanser, 2003); Mike Crang and Nigel Thrift, Thinking space (Routledge, 2000); and Henri Lefebvre, The Production of Space (Blackwell, 1991).

[37] Gilles Deleuze and Felix Guattari, A Thousand Plateus: Capitalism and Schizophrenia (University of Minnesota Press, 1987)

[38] Ibid, p. I I 2

[**39**] Greg Lynn, Folds, Bodies & Blobs (La lettre Volée, 1998)

[40] Greg Lynn, Animate Form (Princeton Architectural Press, 1999)

[41] Harry Francis Mallgrave, T he Architect's Brain: Neuroscience, Creativity, and Architetcure (Wiley-Blackwell, 2010)

[35] Lars Behrisch, Vermessen,

Zählen, Berechnen: Die politische Ordnung des Raums

im 18. Jahrhundert

(Campus, 2006), p. 7

[32] William Mitchell, The Logic

Computation, and Cognition

of Architecture: Design,

(MIT Press, 1990)

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